

Bidding rings and the winner's curse

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This article extends the theory of legal cartels to affiliated private value and common value environments. We show that efficient collusion is always possible in private value environments, but may not be in common value environments with a binding reserve price. In the latter case, collusion does more than simply transfer rents from the seller to the buyers, it also gives buyers a chance to pool their information prior to trade and make an efficient investment decision. However, full efficiency may not be compatible with information revelation. Buyers with high signals may be better off if no one colludes, leading to inefficient trade. This result provides a possible explanation for the low incidence of joint bidding, especially on marginal tracts, in U.S. federal government offshore oil and gas lease auctions.

1. Introduction

■ Collusion in an auction market occurs when a group of bidders takes actions to limit competition among themselves. Colluding bidders are often called a ring, which can include all of the bidders or some subset. There is evidence of collusion in many auction markets. Examples include highway construction contracts (Porter and Zona, 1993), school milk delivery (Pesendorfer, 2000; Porter and Zona, 1999), and timber auctions (Baldwin, Marshall, and Richard, 1997). Collusion is not too surprising because noncooperative behavior is not jointly optimal for bidders. Bidders are collectively better off colluding and transferring gains from trade from the seller to the ring. The problems that a ring faces in dividing the collusive surplus are detection by authorities or by the seller, internal enforcement, entry, and private information about the gains from trade. Legal rings do not have to worry as much about detection or enforcement. An important obstacle that they face is providing incentives to elicit each member's private information about the gains to trade. This raises the following question: can a legal ring collude

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efficiently and still offer its members expected payoffs that exceed what they can earn if the ring does not operate?

The above question has been studied using the tools of mechanism design for auctions with independent private values. These are auctions in which each buyer's valuation depends only upon her own information. The main conclusion of this literature is that bidding rings can collude efficiently and make their members better off. Graham and Marshall (1987) analyze collusion in second-price sealed-bid and English auctions. They show that a second-price knockout auction tournament operated by an outside agent hired by the ring can implement efficient collusion by any subset of *ex ante* identical bidders. In a knockout auction, which is held prior to the seller's auction, the members of the ring bid for the right to be the sole serious cartel bidder, and the winner makes side payments to the other participants based on the bids submitted. Graham and Marshall's proposed mechanism satisfies *ex ante* budget balance but not *ex post* budget balance. Mailath and Zemsky (1991) study second-price auctions with heterogeneous bidders and establish that efficient collusion by any subset of bidders is possible. McAfee and McMillan (1992) study first-price sealed-bid auctions and show that, if the ring includes all bidders, then efficient collusion with *ex post* budget balancing is possible, but it requires transfers to be paid from the member with the highest valuation to those with lower valuations. They assume that bidders commit to the ring before they obtain their private information so that the relevant participation constraints are *ex ante*.

Our primary objective in this article is to study ring formation in first-price sealed-bid auctions of common value assets. The motivation for our study is to explain the incidence of joint bidding in U.S. federal auctions of oil and gas leases in the Outer Continental Shelf (OCS) off the coasts of Louisiana and Texas during the period 1954–1970, inclusive, when joint bidding ventures were legal for all firms.¹ Common value assets are assets where each buyer's valuation depends upon the information of all of the buyers. The canonical example of such assets are oil and gas leases.² There is a component of the value of a tract that is common to all bidders, associated with the size of the deposit. However, bidders may have different information about deposit size and their development costs may differ.

Collusion would appear to be easier to achieve in common value auctions. Competition in auctions of common value assets can lead to inefficient trade: too much trade occurs whenever a buyer bids and his valuation conditional on all of the private signals is less than the asset's acquisition and investment costs, and too little trade occurs whenever no one bids and at least one of the buyers would be willing to do so if he knew all of the private signals. Collusion gives the bidders an opportunity to pool their information prior to trade. In a private value auction, competition results in efficient trade because buyers bid whenever their value exceeds the reserve price, and their value is independent of their rivals' signals. Hence the asset is sold if and only if at least one agent's value exceeds the reserve price. (The competitive bidding outcome might not be efficient if buyers adopt asymmetric strategies, because the asset may then not be sold to the bidder with the highest valuation.) Thus, in contrast to private value auctions, collusion in common value auctions can do more than just transfer rents from the seller to the buyers, it can also permit efficient trade. Buyers in pure common value auctions are also more likely to achieve a consensus on the value of the asset. Indeed, McAfee and McMillan (1992) argue that the reason for their focus on private value auctions is that the optimal ring mechanism in the pure common value case is too simple. Efficiency is attained regardless of which member gets the right to bid in the seller's auction. Thus, an all-inclusive ring can use some exogenous method to allocate the right to one of its members, such as a random allocation with equal probability weights, and ask each bidder to report his information. Bidders have no incentive to misrepresent

¹ In late 1975, concerns over bidding collusion caused Congress to pass legislation prohibiting joint bids involving two or more of the eight largest private oil firms (Exxon, Gulf, Mobil, Shell, Standard Oil of California, Standard Oil of Indiana, Texaco, and British Petroleum) on federal leases on the OCS.

² See Hendricks, Pinkse, and Porter (2003) for evidence that is consistent with the claim that oil and gas leases are common value assets.

their information, and the winner can determine on the basis of the pooled information whether the asset is worth acquiring.

We show, however, that information revelation may prevent rings from forming in common value auctions. The problem lies not in the incentive and budget balance constraints but in the buyers' interim participation constraints. When buyers compete in the auction using interim beliefs, the efficiency of the ring works to the advantage of buyers with low signals, but against a buyer with a high signal and, as a result, the latter may refuse to join the ring. More precisely, buyers with high signals may be able to earn more in the seller's auction than in a ring that uses an efficient, incentive compatible mechanism to allocate the right to bid in the auction. The intuition for this result is as follows. In an efficient ring mechanism, a buyer with a low signal does not have to worry about the winner's curse and is therefore more aggressive in demanding payment for revealing his private signal. As a result, a buyer with a high signal ends up paying less to the seller but more to the other buyers.

Our inefficiency result depends critically upon the assumption that the buyers either do not learn anything about each other's information from the implementation of the ring mechanism, or they commit not to use this information. This is the standard assumption in the literature discussed above. However, Cramton and Palfrey (1995) have pointed out that this may be an unreasonable assumption in many environments. They consider an alternative model of participation constraints in which buyers are allowed to learn from disagreement. Buyers first simultaneously choose to vote for or against the ring mechanism in the interim stage. The mechanism is implemented if it is unanimously ratified; otherwise the buyers bid competitively in the seller's auction under revised beliefs that satisfy a consistency condition that Cramton and Palfrey call ratifiability. In this two-stage game, a buyer's veto decision is a signal about his type. We show that, in a pure common value environment, any ring mechanism that offers all types positive expected profits is unanimously ratifiable. Thus, if buyers are required to make informative veto decisions, our model predicts that efficient collusion can occur in a pure common value first-price auction. In contrast, Tan and Yilankaya (2007) have shown that efficient ring mechanisms are not ratifiable in second-price auctions when values are private and participation is costly.

The two different types of beliefs described above, and hence the relevant participation constraints, lead to quite different outcomes. We argue below that in our application to offshore oil and gas auctions, the relevant participation constraints are based on buyers having interim beliefs in the auction. In other contexts, it may be more reasonable to assume that beliefs adapt. The appropriate model of the participation constraints facing a bidding coalition depends on the environment being considered.

The remainder of this article is organized as follows. In the next section we present a theoretical model that is motivated by the setting of federal offshore oil and gas auctions and show that a modified version of the first-price knockout auction is *ex post* efficient and *ex post* budget balanced. We then describe bidders' interim participation constraints. Bidders' continuation payoffs in the event that a ring does not form may be based on passive beliefs with respect to their rivals' signals, or beliefs may be updated. In Section 3, we study collusion under passive beliefs. We show that if there are private values, there is no information sharing effect and efficient collusion is possible. However, if there are common values and a binding reserve price, information sharing works against collusion. We show that the first-price knockout auction always satisfies the interim participation constraints if values are private but may not do so when values are common. We characterize the set of all incentive compatible, efficient collusive mechanisms for common value environments with independent signals. We show that the first-price knockout auction is an optimal collusive mechanism but that it may not satisfy the interim participation constraints. We provide a necessary and sufficient condition for the constraints to fail and investigate this condition using an example. In Section 4, we examine the effect of buyers learning from disagreement on their incentive to collude. As described above, we focus on pure common value environments in this section, and we show that efficient collusion is possible if there is information leakage. In Section 5, we apply the analysis to study the incidence of joint bidding in

auctions of federal oil and gas leases in the Outer Continental Shelf. Section 6 provides concluding remarks.

2. The model

■ The seller sells an asset using a first-price sealed-bid auction with a preannounced reserve price. Let r denote the sum of the reserve price and any postsale investment that is required to realize the value of the asset. Because we will not study partial rings, we can without loss of generality assume there are only two buyers, labelled $i = 1, 2$. This restriction simplifies the notation considerably.

We denote buyer i 's private signal on the asset by S_i . The signals are real valued and their support is normalized to be the unit interval. Let V denote the unknown component that is common to all buyers' valuations.

Assumption 1. (V, S_1, S_2) are affiliated and symmetric in (S_1, S_2) .

Let F denote the cumulative distribution function of (V, S_1, S_2) with support $[\underline{v}, \bar{v}] \times [0, 1]^2$. Here F is assumed to have a continuous density f . Let $F(s_j | s_i)$ denote the conditional distribution of the signal of buyer i 's rival given $S_i = s_i$. The value of the asset to buyer i is given by $u(V, S_i)$ where u is nonnegative, continuous, and increasing in both arguments. The buyer's utilities depend upon the common component in the same manner, and each buyer's utility is also allowed to depend upon its own private information. Laffont and Vuong (1996) refer to this model as the affiliated values (AV) model. It was first introduced by Wilson (1977) and is a special case of the general symmetric model of Milgrom and Weber (1982). In the AV model, the signals of the other buyers affect the expected utility of buyer i through their affiliation with V and S_i , but they do not enter as an argument of the utility function.

The affiliated values model captures most of the special cases that have been considered in the literature. It includes the case of pure common values (CV), in which each buyer's valuation depends only upon the common factor (i.e., $u(V, S_i) = V$). It also includes the case of private values (PV), in which a buyer's valuation depends only its own signal (i.e., $u(V, S_i) = S_i$). Within the private and common values bidding environments, we will sometimes distinguish between independent signals (IPV and ICV, respectively) and affiliated signals (APV and ACV, respectively).³ Finally, it includes a class of models that have recently received attention, in which the common factor can be expressed as a (deterministic) function of the buyer signals, $V = g(S_i, S_j)$, where g is symmetric, increasing, and continuous. Define $v(S_i, S_j) = u(g(S_i, S_j), S_i)$. Then the restrictions on u and g imply that

$$s_i \geq s_j \implies v(s_i, s_j) \geq v(s_j, s_i)$$

for all $i, j, j \neq i$. If equality holds for all possible signals, then the model is one of pure common values. More generally, each buyer's valuation can be expressed in terms of a common component and a private component. For example, Bulow and Klemperer (2002) assume that $V = S_1 + S_2$ and $v(S_i, S_j) = (1 + \alpha)S_i + S_j$, where $\alpha > 0$.

We model collusion as a problem in mechanism design. The ring must decide whether or not to acquire the asset at cost r and how to divide the collusive surplus. Buyers can make binding commitments to the ring at the interim stage, and side payments are feasible. The collusive mechanism determines which members get the exclusive right to bid in the seller's auction, and any transfers among members. We are interested in mechanisms that satisfy budget balance and efficiency *ex post*. A ring mechanism is *ex post budget balanced* if the sum of the transfers is zero. It is *ex post efficient* if (i) the buyer with the highest signal (and hence the highest valuation) is given the exclusive right to purchase the asset and (ii) he does so if and only if the expected value

³ The theoretical literature classifies auctions in terms of the "reduced-form" valuation $w(s_i, s_{-i}) = E[u_i | S_i = s_i, S_{-i} = s_{-i}]$ rather than the primitives. Values are private if $w(s_i, s_{-i}) = s_i$ and interdependent otherwise. Common values is a special case of interdependent values.

of the asset conditional on the signals of all buyers exceeds r . Condition (ii) is the distinguishing feature of common value environments. In private value environments, each buyer knows whether or not he is willing to pay at least r for the asset. In common value environments, buyers need to share their information to determine whether or not their willingness to pay exceeds r . The pooling of information is especially valuable if r is large. Note that condition (i) is not necessary in the case of pure common values. If all buyers value the asset equally conditional on the same information, then any allocation satisfying condition (ii) is *ex post* efficient. When the buyers do not collude, the equilibrium payoffs of the seller's first-price sealed-bid auction determine their participation constraints.

One class of mechanisms that rings have used to allocate the exclusive right to bid in the seller's auction is knockout auctions. McAfee and McMillan (1992) have shown that a first-price knockout auction implements the optimal collusive mechanism in the IPV environment. In this auction, each member submits a sealed bid. The member with the highest bid is awarded the exclusive right to acquire the asset at cost r from the seller, and pays his bid to the "losing" buyer. Ties are resolved by randomization. We will consider this mechanism but add the requirement that the losing buyer reports his signal to the winning buyer. The winning buyer then updates his beliefs about the value of the asset, and purchases it from the seller at price r if and only if the expected value of the asset conditional on his signal and the reported signal of the losing buyer exceeds r .

Define

$$w(s, t) = E[u(V, S_i) | S_i = s, S_j = t]$$

as buyer i 's expected value of the asset conditional on the event that his signal is equal to s and buyer j 's signal is equal to t . Assumption 1 implies that w is increasing in both arguments. We assume that r is less than $w(1, 1)$, the highest possible valuation. It will also be convenient to normalize payoffs so that $w(0, 0) = 0$.

In the ring, a bidder learns his rival's signal *before* he has to decide whether or not to pay r to the seller. Let b denote the cutoff signal below which a buyer does not bid in the knockout auction. It is defined as $w(b, b) = r$. The interpretation of b is that it is the *lowest* signal at which a buyer can win the knockout auction (i.e., $t < b$) and be certain that the asset is not worth purchasing, conditional on all of the available information. At any higher signal, a buyer is willing to pay a positive amount for the right to purchase the asset at price r because there is some chance that, after winning and learning the other buyer's signal, his valuation exceeds r . Because $w(s, s)$ is increasing, b is unique.

Suppose that in the knockout auction both buyers use a symmetric, increasing bid strategy $B^K(s)$ with boundary condition $B^K(b) = 0$. It is straightforward to show that equilibrium profits are

$$\pi^K(s) = \int_0^s \max\{w(s, t) - r, 0\} dF(t | s) - B^K(s)F(s | s) + \int_s^1 B^K(t) dF(t | s), \quad (1)$$

where

$$B^K(s) = \frac{1}{2} \int_b^s \max\{w(t, t) - r, 0\} dL^K(t | s)$$

and

$$L^K(t | s) = \exp\left(-\int_t^s \frac{2f(x | x)}{F(x | x)} dx\right).$$

It is easily checked that B^K is strictly increasing on the interval $[b, 1]$. For $s < b$, we define $B^K(s) = 0$. The expected payoff to a ring member is strictly positive and constant for s less than b , and strictly increasing in s above b . The payoffs in equation (1) consist of three terms. The first two terms reflect payoffs in the event the member wins the knockout auction, and the third reflects expected payments when the rival wins the knockout.

Because the loser’s report does not affect his payment, he has no reason not to tell the truth. In fact, the only circumstance in which he needs to report his signal is when it is less than b . Otherwise, the winning buyer can infer his signal from his bid. Symmetry and monotonicity imply that the mechanism selects the buyer with the highest signal provided it exceeds b . Ties occur if both buyers submit a bid of zero, but in that case it does not matter who is selected, because neither buyer wants to purchase the asset. The selected buyer purchases the asset if and only if $w(s, t)$ exceeds r . The transfers among the buyers sum to zero by definition. We have therefore established the following result.

Lemma 1. The first-price knockout auction with information sharing is an *ex post* efficient, incentive compatible mechanism that satisfies *ex post* budget balance.

It is worth emphasizing the role of symmetry in the above lemma. The first-price knockout selects the buyer with the highest signal, but efficiency requires that the ring select the buyer with the highest valuation conditional on all of the private signals. In symmetric models, these two criteria are equivalent, and hence information sharing creates no incentive problems.

We assume that buyers form a collusive ring *after* they have acquired their private signals. Hence, the relevant participation constraints are interim: for every possible realization of the signal, a buyer must expect to be at least as well off colluding with the other buyer as he is bidding on his own. The buyer’s continuation payoffs if he chooses not to participate in the ring depends upon whether the failure of the ring to form conveys information about the buyers’ private information. One possibility is to assume that buyers do not update their beliefs following disagreement and bid in the first-price auction using interim beliefs. The literature refers to this case as *passive beliefs*. Another possibility is that buyers learn from disagreement and update their beliefs accordingly. In what follows, we will consider both cases.

3. Collusion under passive beliefs

■ In this section, we study the extent to which an *ex post* efficient, incentive compatible, and budget balanced ring mechanism satisfies the participation constraints under passive beliefs. We first consider general environments with affiliated signals and investigate the conditions under which a first-price knockout auction with information pooling generates an *ex post* efficient allocation that satisfies *ex post* budget balance and the participation constraints. An example is provided to illustrate the circumstances under which a bidding ring is likely to form. We then restrict our attention to auction environments with independent signals where we can exploit the revelation principle and study collusive direct revelation mechanisms. We identify and characterize conditions under which the *ex post* efficiency, *ex ante* budget balance, and participation constraints are compatible for *any* indirect mechanism.

We begin by characterizing the buyers’ continuation payoffs in the event that the ring fails to form and buyers bid individually and noncooperatively in the seller’s first-price auction. We will refer to the latter auction as the *status quo mechanism*. Suppose both buyers use a symmetric bidding strategy $B(s)$ with boundary condition $B(a) = r$ where

$$a = \inf \left\{ s \mid \int_0^s w(s, t) \frac{f(t | s)}{F(s | s)} dt \geq r \right\}.$$

Here a is the cutoff signal below which the buyer does not believe the asset is worth r conditional on winning, in which case the rival bidder has a lower signal. It is straightforward to show that equilibrium profits to a buyer in the seller’s auction are given by (Milgrom and Weber, 1982)

$$\pi^{NC}(s) = \int_0^s w(s, t) f(t | s) dt - B(s)F(s | s), \tag{2}$$

where

$$B(s) = rL(a | s) + \int_a^s w(t, t) dL(t | s)$$

and

$$L(t | s) = \exp\left(-\int_t^s \frac{f(x | x)}{F(x | x)} dx\right).$$

Note that $\pi^{NC}(s)$ is equal to zero for $s < a$ and increasing for $s > a$.

When valuations are private, the buyer’s purchasing decision is contingent only on his own valuation: he bids in either auction if and only if his valuation exceeds r , which implies that $a = b$. This is not the case when valuations have a common component and the reserve price is positive. In that case, we obtain

$$w(b, b) = r = \int_0^a w(a, t) \frac{f(t | a)}{F(a | a)} dt < w(a, a) \implies b < a.$$

Buyers who draw signals between b and a are willing to bid a positive amount in the knockout auction but are not willing to bid in the *status quo* mechanism. There is an option value associated with learning the rival signal after winning the knockout auction. As we shall see, buyers with low signals bid more aggressively in the knockout auction. Buyers with high signals may then prefer the *status quo* mechanism.

Consider first buyers with signals below a . They earn a positive payoff in the knockout auction and zero in the *status quo* mechanism. Clearly, they are better off in the coalition. Thus, if $\pi^{NC}(s)$ exceeds $\pi^K(s)$ at higher signals, then the slope of $\pi^{NC}(s)$ must exceed the slope of $\pi^K(s)$ at any s such that $\pi^K(s) = \pi^{NC}(s)$.

In order to compare the payoffs for buyers with signals above a , we need the following technical lemma. Subscripts denote partial derivatives.

Lemma 2. $A(s, t) \equiv \frac{f_2(t | s)}{f(t | s)} - \frac{F_2(s | s)}{F(s | s)} \geq 0$ for all $s \leq t$.

Lemma 2 is an implication of affiliation and its proof is relegated to Appendix A. Note that, if signals are independent, then $F_2(s | t) = f_2(t | s) = 0$, which in turn implies that $A(s, t) = 0$.

Our next lemma compares the slopes of the equilibrium profit functions at signals above a .

Lemma 3. For any $s > a$,

$$\begin{aligned} & \left(\frac{d\pi^K(s)}{ds} - \frac{d\pi^{NC}(s)}{ds} \right) - \frac{F_2(s | s)}{F(s | s)} [\pi^K(s) - \pi^{NC}(s)] \\ &= - \int_0^s \frac{\partial}{\partial s} \min[r, w(s, t)] f(t | s) dt - \int_0^s \min[r, w(s, t)] A(s, t) f(t | s) dt \\ & \quad + \int_s^1 B^K(t) A(s, t) f(t | s) dt. \end{aligned} \tag{3}$$

The proof of Lemma 3 is given in Appendix A. The lemma identifies two competing effects on the relative slopes of the equilibrium profit functions: the information sharing effect and the affiliation of signals effect. The information sharing effect is absent when values are private or the reserve price is zero. In the former case, the buyer’s decision to purchase the asset does not depend upon the signal of the other buyer because $w(s, t) = s$. As a result, $r = a$, which in turn implies that the first term on the right-hand side of equation (3) is zero. The second term also vanishes because

$$\begin{aligned} \int_0^s r A(s, t) f(t | s) dt &= r \int_0^s \left(\frac{f_2(t | s)}{f(t | s)} - \frac{F_2(s | s)}{F(s | s)} \right) f(t | s) dt \\ &= r \left(\int_0^s f_2(t | s) dt - F_2(s | s) \right) = 0. \end{aligned}$$

In the case where r is zero, the buyer is always willing to purchase the asset because $w(s, t)$ is positive. Once again, the value of the first two terms on the right-hand side of equation (3) are zero. The third term is positive by Lemma 2. Hence, the two profit functions cannot cross, which implies that $\pi^K(s)$ exceeds $\pi^{NC}(s)$ for all s . We have therefore established the following proposition.

Proposition 4. Suppose values are private or the reserve price is not binding. Then the first-price knockout auction is an *ex post* efficient mechanism that satisfies *ex post* budget balance and interim participation constraints.

McAfee and McMillan (1992) have shown that buyers can collude efficiently and earn higher payoffs when values are private and independently distributed. Proposition 4 extends both of these results to affiliated private values. Note that, in the IPV case, $\pi^K(s)$ exceeds $\pi^{NC}(s)$ by a positive constant (i.e., the slopes are equal).⁴

The presence of the information sharing effect works against the formation of an all-inclusive ring. Define $\theta(s)$ by

$$w(s, \theta(s)) = r.$$

Because w is increasing in both arguments, we have that $\theta(s) < b$ for $s > b > a$. Thus, the first term on the right-hand side of equation (3) can be expressed as

$$\begin{aligned} \int_0^s \frac{\partial}{\partial s} \{\min[r, w(s, t)]\} f(t | s) dt &= \int_0^{\theta(s)} \frac{\partial}{\partial s} \{\min[r, w(s, t)]\} f(t | s) dt \\ &\quad + \int_{\theta(s)}^s \frac{\partial}{\partial s} \{\min[r, w(s, t)]\} f(t | s) dt \\ &= \int_0^{\theta(s)} w_1(s, t) f(t | s) dt \geq 0, \end{aligned}$$

where w_1 is the partial derivative of w with respect to the first argument. The term is a measure of the value of a rival's information to a bidder with signal s . When the other buyer's signal lies between 0 and $\theta(s)$, the efficient decision is not to purchase the asset from the seller. This outcome is implemented by the ring but not by the *status quo* mechanism.

Now suppose that signals are independent. In this case, $F_2(s | s) = A(s, t) = 0$ and the difference in the slopes of the two profit functions is equal to the expression given above. Because the value of learning a rival's signal is lower for buyers with higher signals (i.e., $\theta(s)$ falls with an increase in s), π^{NC} increases more rapidly with s than π^K . As a result, π^K can intersect π^{NC} at most once at a signal above a , and if it does so, then the participation constraints for buyers with higher signals are violated.

When signals are affiliated, the second term on the right-hand side of equation (3) cannot be signed. It depends on the interaction of the information sharing and affiliation effects. However, F_2 is negative, which implies that affiliated signals tend to narrow the difference in slopes of the two profit functions. Thus, the affiliation effect favors the formation of a ring. The intuition is that bidding in the *status quo* mechanism is relatively more competitive when signals are affiliated.

Why may buyers with high signals prefer to bid competitively for common value assets? The reason is the winner's curse. Fear of the winner's curse causes buyers to bid cautiously in the *status quo* mechanism, and buyers with low signals do not participate. The latter leads to inefficient trade, because no one may bid even though at least one of the buyers would be willing to do so if he knew all of the private signals. (The converse is also true—a buyer may purchase the asset in the *status quo* mechanism when he would not do so if informed of his rival's signal.) In contrast, the ring is efficient. The winning bidder in the knockout auction learns the private signals of the other members, and therefore purchases the asset if and only if his valuation conditional on all

⁴ Mailath and Zemsky (1991) obtain a similar result for second-price auctions.

of the private signals exceeds investment costs. The efficiency of the ring works to the advantage of buyers with low signals but against a buyer with a high signal. A high-signal buyer ends up paying less to the seller but more to the other buyers.

Under the conditions of Proposition 4, the *status quo* mechanism is efficient and hence there is no tradeoff between efficient collusion and individual rationality. In contrast, in common value auctions with a binding reserve price, the *status quo* mechanism is inefficient and efficient collusion may be incompatible with individual rationality. If the ring is willing to sacrifice efficiency, then it can satisfy the interim participation constraints.

Proposition 5. The first-price knockout auction without information sharing is a mechanism that satisfies *ex post* budget balance and interim participation constraints.

The proof of Proposition 5 is given in Appendix A. Note that, as in the case of Proposition 4, the ring uses a mechanism that implements the same trades as the *status quo* mechanism. However, it is difficult to imagine how buyers can enforce a collusive agreement that prohibits them from sharing information when it is *ex post* optimal for them to do so.

An important implication of Proposition 5 is that, if possible, bidders should sign a collusive agreement before they learn their private information. *Ex ante*, the expected cartel payoff always exceeds the expected payoff from competitive bidding. The cartel gains rents from the seller and eliminates inefficiency losses.

□ **Example.** What are the circumstances under which a cartel is likely to form? We study this question using the wallet game (Bulow and Klemperer, 2002; Klemperer, 1998) in which each buyer's value of the asset is the sum of the buyers' signals and the signals are independently distributed. Suppose there are two buyers and the distribution of each buyer's signal is $F(s) = s^q$, where $q > 0$. The parameter q determines the shape of the distribution. A higher value of q shifts probability mass away from lower signals to higher signals. Higher values of q also means more optimistic priors because

$$E(s) = \frac{q}{1+q}.$$

Note that if $q = 1$, signals are uniformly distributed on the unit interval. Preferences of the buyers are identical and given by

$$w(s, t) = s + t.$$

Because own and rival signals are weighted equally, the model is one of pure common values. The value of the reserve price r ranges between 0 and 2. We use r to parameterize the importance of the information sharing effect.

As discussed earlier, the payoff curve under the knockout auction and the payoff curve under the *status quo* mechanism can intersect at most once at a signal above a . This means that we only need to check the participation constraints for buyers with higher signals. In the example, we can easily identify the values of q and r where the participation constraint fails to be satisfied by comparing the equilibrium profits of the highest type. The critical cutoff signal value for participation in the competitive auction is given by

$$a = \frac{1+q}{1+2q}r.$$

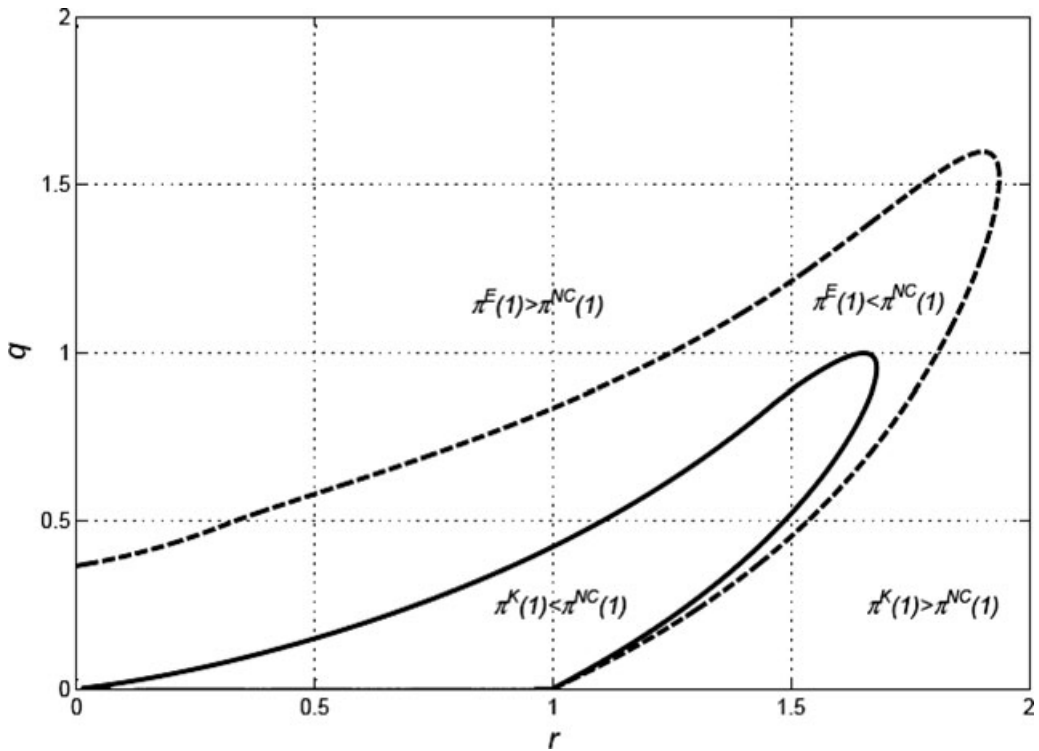
In the knockout auction, it is given by

$$b = \frac{r}{2}.$$

Note that the fraction of types that bid in the knockout but not in the *status quo* mechanism, $a - b$, decreases with q and increases with r . The equilibrium payoff to the highest type in the *status quo* mechanism is

FIGURE 1

RING AND STATUS QUO PAYOFFS FOR THE HIGHEST TYPE



$$\pi^{NC}(1) = \frac{1 - a^{1+q}}{1 + q}.$$

His payoff in the knockout auction is

$$\pi^K(1) = \left(\frac{1}{1 + 2q} \right) \left[\frac{2q^2 + 2q + 1}{(1 + q)} - \left(\frac{r}{2} \right)^{1+2q} \right] - \frac{r}{2} + \frac{\max\{r - 1, 0\}^{1+q}}{1 + q}.$$

Details on the derivations of these equations are given in Appendix B.

Figure 1 compares the two payoffs for the highest type in (r, q) space. The solid curve is the locus of points where the highest type's ring profits are equal to his profits in the *status quo* mechanism. The region below this curve represents the area where the highest type earns more from the *status quo* mechanism than from the ring. For fixed r , a higher value of q means that information sharing becomes less valuable, causing the highest buyer to bid relatively more aggressively in the *status quo* mechanism than the knockout auction. Thus, high values of q favor the ring over the *status quo* mechanism. For fixed q , an increase in r makes information sharing more valuable. This has two effects on the highest type's payoffs. It enhances his strategic advantage in the competitive auction, because the winner's curse is stronger and scares off more types. But it also makes learning the other buyer's signal more valuable. The first effect dominates for low values of r (i.e., $r < 1$) and the second dominates for high values of r (i.e., $r > 1$). The tradeoff between these two effects accounts for the nonmonotonic relationship between q and r .

In our application, rings frequently use an equal-sharing mechanism in which members report their signals and share asset returns equally. This mechanism satisfies *ex post* budget balance. It is also *ex post* efficient in our example because it does not matter which buyer gets the asset under the assumption of pure common values. Finally, it is incentive compatible because

each buyer wants the ring to buy the asset if and only if the expected value of the asset net of acquisition costs is positive. We compare the two mechanisms in Figure 1. The dashed line corresponds to points where the highest type's profits from the equal-sharing ring are equal to his profits in the *status quo* mechanism. Clearly, the region where the equal-sharing mechanism is *not* enforceable is larger than and contains the region where the first-price knockout is not enforceable. The reason is that high types get a larger share of the cartel surplus in the first-price knockout auction than in the equal-sharing mechanism. Thus, the ring is more likely to form when it allocates the exclusive right to bid in the seller's auction with a first-price knockout auction than when an equal-sharing agreement is employed.

In a previous version of this article (Hendricks, Porter, and Tan, 2003), we also examined the effect of more buyers on the participation constraints. The increase in n has two effects. First, the winner's curse effect is strengthened, which enhances the high type's strategic advantage in the *status quo* mechanism but makes information pooling more valuable. When r is low, the strategic advantage is more important and the additional buyer reduces the likelihood that the ring is enforceable. When r is high, information pooling is more important, and the additional buyer makes it more likely for the ring to be enforceable. Second, the level of competition increases. Because the competitive effect is stronger in the *status quo* mechanism than in the ring mechanism, the area in which the ring is not enforceable is reduced.⁵

In summary, the all-inclusive ring is not enforceable when priors are pessimistic (i.e., q is low), acquisition costs are substantial (i.e., r is not too low or too high), and the number of buyers is small.

□ **Independent signals.** The preceding analysis demonstrates that, if information sharing is important, a buyer with a high signal may prefer the *status quo* mechanism to an all-inclusive ring when the ring uses a first-price knockout auction to allocate the option to purchase the asset at price r . One can criticize this result on the grounds that the first-price knockout mechanism is only one of many possible collusive mechanisms; we have not ruled out the possibility of another efficient ring mechanism that does satisfy interim rationality. To show that this is unlikely, we consider a more restrictive environment with independent signals. In this environment, we can exploit the revelation principle and study collusive direct revelation mechanisms. We then prove that the first-price knockout auction implements the optimal cartel mechanism.

In a collusive direct revelation mechanism, the ring's representative in the seller's auction, and side payments between the buyers, are determined as functions of the buyers' reported signals. The mechanism is a pair $\{Q, P\}$, where $Q : [0, 1]^2 \rightarrow [0, 1]^2$ and $P : [0, 1]^2 \rightarrow R^2$. Let x_i denote the report by buyer i . Given reports (x_1, x_2) , the probability that buyer i obtains the right to bid in the seller's auction is $Q_i(x_i, x_j)$ and its expected side payment is $P_i(x_i, x_j)$. Clearly,

$$Q_1(x_1, x_2) + Q_2(x_2, x_1) \leq 1$$

for all $(x_1, x_2) \in [0, 1]^2$. We assume that transfers are feasible if they satisfy

$$P_1(x_1, x_2) + P_2(x_2, x_1) = 0$$

for every pair of reported signals (x_1, x_2) . This requires the ring to balance its budget *ex post*. A weaker requirement is *ex ante* budget balance which only requires that transfers between buyers sum to zero on average.

Suppose buyer j reports truthfully. Then the expected payoff to buyer i with signal s_i and report x_i is

$$\pi_i(s_i, x_i) = E_{s_j} [Q_i(x_i, s_j) \max\{w(s_i, s_j) - r, 0\} + P_i(x_i, s_j)]. \tag{4}$$

⁵ We conjecture that the competitive effect dominates when the expected value of the object is held constant as the number of buyers gets large and that, in the limit, the all-inclusive ring satisfies the participation constraints. For example, it is not difficult to specify common value environments in which a buyer with the highest signal will not want to participate in the *status quo* mechanism as the number of buyers gets large. In these cases, the ring satisfies participation constraints because all buyers make positive profits from collusion.

Denote $\pi_i(s_i, s_i)$ by $\pi_i(s_i)$. A ring mechanism $\{Q, P\}$ is *incentive compatible* if for all $s_i, x_i \in [0, 1], i = 1, 2$,

$$\pi_i(s_i) \geq \pi_i(s_i, x_i).$$

The following standard lemma characterizes the set of incentive compatible mechanisms.

Lemma 6. A ring mechanism $\{Q, P\}$ is incentive compatible if and only if for any $s_i, x_i \in [0, 1]$,

$$\frac{d\pi_i(s_i)}{ds_i} = E_{s_j} \left[Q_i(s_i, s_j) \frac{\partial}{\partial s_i} \max\{w(s_i, s_j) - r, 0\} \right], \tag{5}$$

and

$$E_{s_j} [(\partial Q_i(x_i, s_j) / \partial x_i) \max\{w(s_i, s_j) - r, 0\}] \geq 0.$$

Efficiency implies that the buyer with highest valuation is awarded the exclusive right to acquire the asset at price r and does so if and only if the expected value of the asset conditional on (s_1, s_2) exceeds r . More formally, a direct mechanism is *ex post* efficient if

$$Q_i(s_i, s_j) = \begin{cases} 1 & \text{if } s_i > s_j > \theta(s_i) \\ 0 & \text{otherwise,} \end{cases}$$

where as before $\theta(s)$ is defined by

$$w(s, \theta(s)) = r.$$

Recall that, because w is increasing in both arguments, $\theta(s) < b$ for $s > b$. Combining the incentive compatibility and budget balance with efficiency yields the following characterization of payoffs.

Lemma 7. Suppose signals are independently distributed and $w(s, t) > w(t, s)$ for all $s > t$. Then the payoff to buyer i with signal s in any *ex post* efficient, incentive compatible mechanism that satisfies *ex ante* budget balance is given by

$$\pi_i^C(s) = \pi_{i0} + \int_{\theta(s)}^s [w(s, t) - r] dF(t) - \int_b^s [w(t, t) - r] dF(t)$$

for $s > b$ and is equal to π_{i0} otherwise, where

$$\pi_{10} + \pi_{20} = 2 \int_b^1 [w(t, t) - r][1 - F(t)] dF(t).$$

The proof of Lemma 7 is given in Appendix A. *Ex post* efficiency, incentive compatibility, and *ex ante* budget balance uniquely determine the payoff of each member of the ring up to a constant. In an anonymous mechanism, the buyers are treated symmetrically, which implies that $\pi_{10} = \pi_{20}$. Any indirect, anonymous ring mechanism that is *ex post* efficient and satisfies the stronger restriction of *ex post* budget balance generates identical expected payoffs. It then follows from Lemma 1 that these payoffs can be implemented by the first-price knockout auction with information pooling.

Proposition 8. Suppose signals are independently distributed and $w(s, t) > w(t, s)$ for all $s > t$. Then any *ex ante* budget balanced, *ex post* efficient, incentive compatible, anonymous ring mechanism can be implemented by a first-price knockout auction with information sharing.

Proposition 8 extends McAfee and McMillan’s result for an independent private values model to common value models with independent signals. Of course, the first-price knockout auction is not the only implementable mechanism. A second-price knockout auction also works.

Corollary 9. Suppose signals are independently distributed and $w(s, t) > w(t, s)$ for all $s > t$. Any incentive compatible, *ex ante* budget balanced, *ex post* efficient, ring mechanism satisfies the interim participation constraints if and only if $\pi_i^C(1) > \pi_i^{NC}(1)$ for $i = 1, 2$.

Corollary 9 follows from Lemma 7 and its proof is given in Appendix A. Corollary 9 establishes a useful necessary and sufficient condition for efficiency, incentive compatibility, and budget balance to conflict with the interim participation constraints. A similar example to the one in the first subsection of Section 3 can be easily provided. It can be shown that the payoffs in the knockout auction and the *status quo* mechanism in the wallet game example are continuous in the weights assigned to own and rival signals. Hence, if buyers weight their own signal more heavily than their rival's signal, then the example can also be used to illustrate how the necessary and sufficient conditions of Corollary 9 may not be met.

It is worth noting that in a pure common value environment with independent signals, efficiency does not require that the buyer with the highest signal win the asset. *Ex post* efficiency is attained regardless of which buyer wins the asset, as long as the buyers report their private signals. In this case, a weak ring, which McAfee and McMillan define as a ring that cannot make transfer payments, can be efficient. The mechanism that awards the right to purchase the asset randomly to one member, and all other members report their private signals, is efficient and incentive compatible. Note, however, that this mechanism does not generate the same payoffs as the first-price knockout auction. The indeterminacy of the allocation rule implies that efficiency, incentive compatibility, budget balance, and anonymity do not uniquely determine the payoffs to ring members. Nevertheless, under certain conditions identified in Proposition 10, we can show that the joint payoffs of the buyers at the highest signal are maximized when an "efficient" allocation rule (in the sense that the buyer with the higher signal receives the object) is used. The intuition is that the "efficient" allocation rule favors the buyer with higher signals. Because the knockout auction is such an "efficient" allocation rule, if the knockout auction fails to satisfy the participation constraints, then other mechanisms will also not satisfy the participation constraints.

Proposition 10. Suppose signals are independently distributed, $w(s, t) = w(t, s)$, and

$$w_1(s, t) \frac{F(s)}{f(s)} \geq w_1(t, s) \frac{F(t)}{f(t)}$$

if and only if $s \geq t$. Then any incentive compatible, *ex ante* budget balanced, *ex post* efficient, ring mechanism fails to satisfy the interim participation constraints if $\pi_i^K(1) < \pi_i^{NC}(1)$ for $i = 1, 2$.

Proposition 10 establishes a useful sufficient condition for checking whether an indirect mechanism such as the equal-sharing mechanism conflicts with the participation constraints. If the highest type obtains a higher payoff from bidding competitively than from colluding when the ring uses a first-price knockout auction, then it also prefers the *status quo* mechanism to a ring that uses the equal-sharing mechanism, or any other efficient, incentive compatible, budget balancing mechanism.⁶

In the wallet game example in Section 3, $w_1(s, t) = w_1(t, s) = 1$ and $F(s)/f(s)$ is increasing, so that the first set of conditions in Proposition 10 is satisfied. The example illustrates how the sufficient condition in Proposition 10, $\pi_i^K(1) < \pi_i^{NC}(1)$, may be met.

4. Collusion with information leakage

■ The main conclusion of the previous section is that cartels are likely to form when valuations are private but that high types may have an incentive to deviate from collusion when valuations

⁶ Because the seller's revenue in the sealed-bid, second-price auction and sealed-bid, first-price auction are the same when signals are independent, this proposition also applies to cases where the seller uses a sealed-bid, second-price auction.

are common. However, this result relied upon the assumption that buyers did not learn from disagreement and bid in the *status quo* mechanism using interim beliefs. We now relax this assumption and allow buyers to revise their beliefs about each other's type if one or both buyers refuse to join the ring. Cramton and Palfrey (1995) refer to this issue as the information leakage problem. The revision in beliefs will affect the buyers' bidding behavior and their equilibrium payoffs from disagreement, and hence their incentive to participate in the ring mechanism.

Following Cramton and Palfrey, we shall assume buyers play a two-stage veto game at the interim stage. In the first stage, both buyers simultaneously vote for or against the ring mechanism based on their private signals. The ring mechanism is selected by an uninformed third party so no information is revealed by its selection. If both buyers vote for the mechanism, then it is implemented. If the ring does not form, then buyers participate independently in the auction knowing the votes of both buyers. A pure strategy for buyer i in the first stage of the game is a binary function that is equal to 1 if bidder i votes for the ring and 0 otherwise. Let $h \in \{(0, 0), (1, 0), (0, 1)\}$ denote a voting outcome in the event of a veto. Given outcome h , the two buyers are bidding competitively in a first-price sealed-bid auction in which buyer i believes that buyer j 's type is drawn from a distribution $F_j(\cdot | h)$. This distribution is well defined and derived from F using Bayes rule whenever h occurs with positive probability.

We are primarily interested in determining whether the pair of voting strategies in which both buyers are certain to vote for the ring can be an equilibrium. In this case, beliefs following a veto are not well defined. We could exploit this freedom and specify beliefs which ensure that all types want to vote for the ring.⁷ However, such beliefs may not be reasonable. Instead, we adopt the consistency requirement proposed by Cramton and Palfrey (1995).⁸ They suppose that, if bidder i vetoes the mechanism, then bidder j tries to rationalize i 's decision by identifying a set of types for i that could have benefited from the veto. Let V_i denote the set of buyer i types that veto. Given their payoffs from the ring mechanism, a veto set V_i is credible if there is an equilibrium in the auction with updated beliefs that gives higher payoffs to every type in V_i and lower payoffs to every type not in V_i .⁹ In other words, types in a credible veto set have an incentive to veto the cartel mechanism if doing so makes the rival bidder believe that they belong to this set, which in turn justifies the belief. The ring mechanism is ratifiable if there is no credible veto set for either bidder.

The main difficulty with applying this definition to our game is characterizing equilibria of a first-price sealed-bid auction for arbitrary veto sets and computing the associated payoffs. However, in the special case of pure common values, the following result can be established without computing the equilibrium.

Proposition 11. Suppose $u(V, s_i) = V$. Then any ring mechanism that gives buyers positive interim payoff is ratifiable.

The intuition for Proposition 11 is as follows. If buyer i deviates and vetoes the ring mechanism, then every type in a credible veto set V_i must earn strictly positive profits in the *status quo* mechanism because all types earn positive profits from participating in the ring. Hence, each veto type must bid at least r and win with positive probability. The latter condition implies that there exists a nondegenerate set of buyer j types, R_j , who are willing to submit bids that are certain to lose. But, given the common value assumption, the expected value of the object conditional on winning at bids near the lower bound of the veto bids must be higher for the higher types in R_j than for the veto types bidding in this range. The veto types are averaging across all types in R_j whereas the high types in R_j know that they are above average. Hence, the high types can earn positive expected profits by bidding more aggressively, above the lower bound of

⁷ For example, we could specify that if player j deviates and vetoes the ring, then player i believes that player j 's type is the highest type. Given this belief, player j cannot make positive profits bidding in the auction.

⁸ The refinement is based on Grossman and Perry (1986). See Cramton and Palfrey for a detailed discussion on the relationship of their refinement and other refinements that restrict off-equilibrium path beliefs.

⁹ There is no restriction for types that are indifferent between vetoing and not.

the bids of the veto types. This argument implies that the veto type(s) submitting the lowest bid earns zero profits in the *status quo* mechanism, which leads to a contradiction.

The proposition establishes that unanimous ratification is an equilibrium to the two-stage game. Thus, when buyers learn from disagreement, collusion is an equilibrium outcome, at least in pure common value models. The veto is a positive signal of asset value and causes the rival buyer to bid more aggressively, discouraging bidders from vetoing the ring. We have investigated the issue of uniqueness but have not been able to rule out equilibria in which some types of both players veto and other types do not, at least not without imposing strong restrictions on the game. Characterizing the set of equilibria is beyond the scope of this article.

5. Application

■ The motivating example for our study is joint bidding in U.S. federal auctions of wildcat oil and gas leases in the Outer Continental Shelf off the coasts of Louisiana and Texas during the period 1954–1970. These auctions satisfy the assumptions of our *status quo* model. The tracts are sold using a first-price, sealed-bid auction. The announced reserve price for tracts in our sample is \$15 per acre but the postsale investment required to realize the value of a lease is substantial, approximately one to two million dollars per tract. Prior to the sale, firms are allowed to conduct seismic studies on the tracts, but they are not permitted to drill exploratory wells. The seismic studies yield noisy private signals about the value of the tracts. The size of any oil or gas deposit on the tract represents the common component of tract value, and the bidder-specific components are exploration and drilling costs. The variation in the value of the deposit is many times larger than the variation in costs, which implies that, to a first approximation, the auction is a pure common value auction. Firms are symmetric in the sense that they are equally informed, although they may have quite different opinions about the likelihood of finding oil and gas, depending upon the content and analyses of the surveys. The potential gains from joint bidding appear to be substantial. The stakes are large, and the risks significant. By pooling geological data and expertise in interpreting the data, firms could reduce the risk of buying dry leases and, by pooling financial resources, they can bid for more leases and diversify away more of the tract-specific uncertainties.

These auctions also satisfy the model's assumptions regarding collusion. All firms were allowed to bid jointly prior to 1975, and joint bidding agreements are enforced by legally binding contracts.¹⁰ To ensure commitment, the agreements typically require each participant to offer their partners equal ownership shares if it wins the tract with a bid that is not sanctioned by the ring. This provision eliminates the incentive participants may otherwise have had to pretend disinterest in a tract and then outbid the ring. The collusive mechanism was often equal division (Hendricks and Porter, 1992), which is consistent with a pure common value environment. Most joint bidding agreements are area and sale specific, that is, firms that bid jointly in one area of the sale did not necessarily do so in other areas, nor did they bid jointly in other sales. Joint bidding agreements are typically struck shortly before the sale date and *after* the firms have acquired their private information about the tracts. We suspect that *ex ante* bidding agreements were not prevalent because firms were reluctant to divulge their methods of interpreting seismic data.

Firms were invited to attend a public meeting if they were interested in participating in a joint bidding venture. This procedure would seem to imply that the relevant participation constraints

¹⁰ The procedure is described by Mobil Oil Corporation in its testimony submitted on February 19, 1976, to the House of Representatives, Subcommittee on Monopolies and Commercial Law of the Committee on the Judiciary:

The bidding groups are formed under a bidding agreement, a formal written document executed by all parties prior to any discussions relating to bonus values. This agreement establishes procedures for arriving at a joint bid and provides for the protection of each individual company in the event agreement cannot be reached. The agreement can either cover the entire sale area or, more commonly, be limited to a specific area of mutual interest (AMI) to the companies involved.

TABLE 1 Bidding Behavior Conditional on Level of Big12 Competition

| <i>NPot12</i> | Tracts | Mean high bid (\$) | Bids per tract | Big12 bids All bids (%) | Joint Big12 All Big12 (%) |
|---------------|--------|--------------------|----------------|-------------------------|---------------------------|
| 0 | 14 | 434,425 | 1.21 | 0.0 | NA |
| 1 | 110 | 1,480,887 | 1.23 | 74.8 | 0.0 |
| 2 | 74 | 2,315,983 | 1.69 | 69.6 | 3.4 |
| 3 | 130 | 2,360,185 | 1.92 | 80.3 | 8.0 |
| 4 | 136 | 2,936,417 | 2.32 | 70.3 | 14.0 |
| 5 | 70 | 4,011,824 | 2.47 | 76.9 | 16.5 |
| 6 | 52 | 4,502,450 | 3.42 | 82.6 | 19.7 |
| 7 | 77 | 5,451,971 | 4.03 | 71.6 | 18.0 |
| 8 | 159 | 5,936,896 | 3.49 | 78.9 | 19.4 |
| 9 | 132 | 8,007,384 | 4.42 | 73.8 | 20.5 |
| 10 | 128 | 9,136,267 | 5.75 | 75.3 | 20.6 |
| 11 | 113 | 10,487,946 | 5.68 | 73.5 | 23.5 |
| 12 | 65 | 21,815,490 | 8.32 | 66.0 | 24.1 |
| Total | 1260 | 6,154,622 | 3.62 | 73.8 | 18.6 |

NA, not applicable.

are the ones derived under the assumption of learning, in which case our model would predict a very high incidence of joint bids. We find that the opposite is true: solo bidding was the dominant form of bidding for the most active participants. Joint bids involving pairs of the most active firms represented less than 15% of all their bids, even though joint bidding agreements were legal. Furthermore, if these firms bid jointly, they almost always did so in pairs, and not in all-inclusive partnerships.

The learning model would apply if firms vote for or against a joint bid on an individual tract. The joint venture agreement, however, covers blocks of tracts, typically 25–50 tracts, and not individual tracts. Thus, firms had to choose between bidding jointly on all of the tracts or on none of them. They did not have the option of making this choice tract by tract. Consequently, if a bidder refuses to join the ring, then the other bidders could infer that the nonparticipating bidder has obtained favorable information about one or more tracts in the area, but they do not know which tracts and, because most are not worth bidding for, the inference is likely to have little impact on beliefs about individual tracts. Thus, the standard model of participation constraints (i.e., passive beliefs) seems to be a better approximation of the firms' joint venture decisions in OCS auctions than the model with information leakage.

Given this hypothesis, the prediction that we wish to examine is whether joint ventures among major bidders are unlikely to occur on marginal tracts with a small number of competitors. We examine more valuable tracts with more competitors as a standard of comparison. Our sample consists of the nine sales of wildcat tracts off the coasts of Texas and Louisiana during the period 1954–1970 inclusive, in which a total of 1260 tracts received bids. Table 1 provides summary statistics on bidding behavior of the 12 most active bidders (Big12) in our sample. The tracts are classified by the number of potential Big12 bidders, *NPot12*, which ranges from 0 to 12. This number is constructed for each tract based on its location and who bid in the area around tract *t*, and when. For tracts that were drilled, location is identified by the longitudinal and latitudinal coordinates of the well. Tracts that were not drilled are assigned coordinates by interpolation from nearby tracts that were drilled. A *neighborhood for a tract* consists of all tracts whose registered locations are close to the tract and *that were offered for sale at the same time as or before the tract*. Ignoring irregular tract sizes and boundary effects, the maximum possible size of a neighborhood is 25 tracts or 125,000 acres. The number of potential bidders on a tract is the number of Big12 firms that bid on the tract or in its neighborhood, either solo or jointly. The rationale is that if a Big12 firm is interested in the area, then it will probably bid on at least one tract.

For each value of *NPot12*, the second column of Table 1 gives the number of tracts, and the third gives the mean high bid. The mean high bid increases from \$434 thousand on tracts where

none of the Big12 firms are potential bidders to \$21.8 million on tracts where every Big12 firm is a potential bidder. The mean high bid in the sample is \$6.2 million per tract. (Bids are expressed in 1982 dollars.) *ex ante* expectations, as measured by the high bid, are positively correlated with the number of potential bidders.

The last three columns report the average number of bids per tract, the proportion of those bids submitted by Big12 firms, and the proportion of Big12 bids that are joint. Most joint bids involve only two Big12 bidders, which suggests that forming an all-inclusive cartel on tracts with more than two or three Big12 bidders is difficult. Big12 bidders submit 73.8% of all bids, and this proportion is roughly constant in *NPot12*. Because the number of bids per tract is increasing in *NPot12*, the average number of bids by non-Big12 firms increases with the level of Big12 competition. On average, there is less than one non-Big12 bid per tract. The proportion of Big12 bids that were joint is less than 10% on marginal tracts, and is roughly constant at approximately 20% for tracts with more than four Big12 potential bidders. Thus, the incidence of joint bidding is significantly lower on marginal tracts than on more valuable, more competitive tracts. This is true despite the fact that the costs of forming an all-inclusive cartel may be lower on marginal tracts, and the gains from reduced competition and from pooling of information are relatively higher. The average winning bid on marginal tracts was over \$1 million, which is well above the minimum price of approximately \$75,000. The cost of drilling an exploratory well is approximately \$1.5 million, and the risk of drilling a dry well is high, because only 39% of the tracts sold were productive.

In practice, the government could and did reject bids above the stated minimum price. The rejection rate was less than 10% on wildcat tracts. It is conceivable that firms do not submit joint bids on tracts with few potential bidders if they are concerned that their bid will be rejected, if the government reacts to the absence of competition. As Porter (1995) notes, wildcat bids were much more likely to be rejected when there were relatively few bids, and when these bids were low. However, only 3 high joint bids were rejected in our sample, out of 167 high joint bids, or 1.8%, as opposed to 7.9% of high solo bids. When one conditions on the level of the high bid, the rejection rule appears to favor joint bids, if anything. (Of course, this does not prove that, had more joint bids been submitted, they would have been accepted with the same frequency.)

Another explanation for the low incidence of joint bids is that joint bids are not an accurate measure of the incidence of collusive bidding. Indeed, in his congressional testimony in 1976, Darius Gaskins (Gaskins and Vann, 1976) argued that the collusive effects of joint ventures should not be measured solely in terms of joint bids. In negotiating over which tracts to bid jointly, the members of the joint venture could learn more about each others' bidding intentions on other tracts and coordinate their solo bids on these tracts. In a previous version of this article (Hendricks, Porter, and Tan, 2003), we examine this hypothesis and find no evidence that joint bidding in an area affected the participants' contemporaneous bidding behavior outside that area.

6. Conclusion

■ We have shown that the trading inefficiency caused by the “winner’s curse” can be an important obstacle to collusion in auctions of common value assets with a binding reserve price or *ex post* investment. The theory predicts that buyers are unlikely to collude when investment costs are substantial, the number of buyers is small, priors are relatively pessimistic, and beliefs are passive. These conditions are satisfied in the auctions of federal offshore oil and gas leases on marginal tracts. Thus, our theory provides an explanation for the low incidence of joint bids on these tracts, where the relative gains from colluding were large, because there was less competition and information sharing was valuable. Interestingly, the learning model of participation constraints suggests that the situation would have been quite different if the firms had adopted a mechanism in which joint bidding decisions were taken on a tract-by-tract basis. In that case, refusal to bid jointly on a tract would cause beliefs about that tract, and therefore bidding behavior, to change. The information leakage would have encouraged firms to participate in a joint bid.

In our application to offshore oil and gas auctions, most of the Big12 joint ventures consisted of only two firms. An analysis of partial rings in common value environments is an interesting subject for future research. McAfee and McMillan (1992) study this issue in a simple model in which each buyer’s private value is an independent Bernoulli random variable. They show that, in equilibrium, the noncolluding bidder is better off than ring members, and that a ring of at least three bidders always forms. Thus, a ring always forms, but the reduced competition effect can explain partial rings. However, the situation is considerably more complicated when values are affiliated, because the noncolluding bidder is at an informational disadvantage when his rivals form a ring, and this effect could offset the benefit from the reduction in competition. Waehrer and Perry (2003) and Mares and Shor (2008) also study the effects of mergers in second-price auctions in private and common value environments, respectively.

Our results are for legal cartels. However, the analysis may also prove useful for understanding self-enforcing cartels. Athey and Bagwell (2001) and Athey, Bagwell, and Sanchirico (2004) study optimal collusion in markets where firms receive privately observed, i.i.d. cost shocks. The firms can communicate with each other to determine who has the lowest cost, but they cannot make side payments to each other. Their modelling approach is to recast the repeated, hidden information game as a static mechanism, similar to that analyzed in the legal cartel literature. They show that, if firms are sufficiently patient, they can use “market share favors” to implement efficient collusion. Our results suggest that it may be more difficult to collude if cost shocks contain a common component.

Appendix

■ *Proof of Lemma 2.* Affiliation implies that $f_2(t | s)/f(t | s)$ is increasing in t and that $f(t | s)/F(t | s)$ is increasing in s . It follows that, for any $s \leq t$,

$$\frac{f_2(t | s)}{f(t | s)} \geq \frac{f_2(s | s)}{f(s | s)} \geq \frac{F_2(s | s)}{F(s | s)},$$

where the second inequality follows from

$$\frac{\partial}{\partial s} \left(\frac{f(t | s)}{F(f | s)} \right) = \frac{f_2(t | s)}{F(t | s)} - \frac{f(t | s)F_2(t | s)}{F^2(t | s)} \geq 0.$$

Q.E.D.

Proof of Lemma 3. In a first-price auction with a reserve price r , the equilibrium payoff to a buyer with signal $s > a$ is

$$\pi^{NC}(s) = \int_0^s w(s, t)f(t | s) dt - B(s)F(s | s).$$

Differentiating with respect to s and using the envelope theorem, we obtain

$$\frac{d\pi^{NC}(s)}{ds} = \int_0^s \left(w_1(s, t) + w(s, t) \frac{f_2(t | s)}{f(t | s)} \right) f(t | s) dt - B(s)F_2(s | s),$$

where $w_1(s, t)$, $F_2(x | s)$, and $f_2(x | s)$ represent partial derivatives with respect to s . Because

$$B(s) = \frac{\int_0^s w(s, t)f(t | s) ds}{F(s | s)} - \frac{\pi^{NC}(s)}{F(s | s)},$$

it follows that

$$\frac{d\pi^{NC}(s)}{ds} = \frac{F_2(s | s)}{F(s | s)} \pi^{NC}(s) + \int_0^s [w_1(s, t) + w(s, t)A(s, t)]f(t | s) dt,$$

where

$$A(s, t) = \frac{f_2(t | s)}{f(t | s)} - \frac{F_2(s | s)}{F(s | s)}.$$

Equilibrium profits to a buyer with signal $s > b$ in a first-price knockout auction with information pooling is

$$\pi^K(s) = \int_0^s \max\{w(s, t) - r, 0\}f(t | s) dt - B^K(s)F(s | s) + \int_s^1 B^K(t)f(t | s) dt.$$

Differentiation with respect to s and using the envelope theorem, we obtain

$$\begin{aligned} \frac{d\pi^K(s)}{ds} &= \int_0^s \left(\frac{\partial}{\partial s} \{ \max[w(s, t) - r, 0] + \max\{w(s, t) - r, 0\} \frac{f_2(t|s)}{f(t|s)} \} \right) f(t|s) dt \\ &\quad - B^K(s)F_2(s|s) + \int_s^1 B^K(t) \frac{f_2(t|s)}{f(t|s)} f(t|s) dt. \end{aligned}$$

Using the definition of $\pi^K(s)$ yields

$$\begin{aligned} \frac{d\pi^K(s)}{ds} &= \frac{F_2(s|s)}{F(s|s)} \pi^K(s) + \int_0^s \frac{\partial}{\partial s} \{ \max[w(s, t) - r, 0] \} f(t|s) dt \\ &\quad + \int_0^s \max\{w(s, t) - r, 0\} A(s, t) f(t|s) dt + \int_s^1 B^K(t) A(s, t) dF(t|s). \end{aligned}$$

The difference in slopes of the two curves at any $s > a$ is

$$\begin{aligned} \frac{d\pi^K(s)}{ds} - \frac{d\pi^{NC}(s)}{ds} &= \frac{F_2(s|s)}{F(s|s)} [\pi^K(s) - \pi^{NC}(s)] - \int_0^s \frac{\partial}{\partial s} \{ \min[r, w(s, t)] \} f(t|s) dt \\ &\quad - \int_0^s \min[r, w(s, t)] A(s, t) f(t|s) dt + \int_s^1 B^K(t) A(s, t) f(t|s) dt. \end{aligned}$$

Q.E.D.

Proof of Proposition 5. Equilibrium profits to a buyer with signal $s > a$ in a first-price knockout auction without information pooling is

$$\pi(s) = (\bar{w}(s) - r - B(s))F(s|s) + \int_s^1 B(t)f(t|s) dt,$$

where

$$\bar{w}(s) = E[w(s, t) | t < s].$$

Differentiation with respect to s and using the envelope theorem, we obtain

$$\frac{d\pi(s)}{ds} = \bar{w}'(s)F(s|s) + (\bar{w}(s) - r - B(s))F_2(s|s) + \int_s^1 B(t) \frac{f_2(t|s)}{f(t|s)} f(t|s) dt.$$

Using the definition of $\pi(s)$ yields

$$\frac{d\pi(s)}{ds} = \frac{F_2(s|s)}{F(s|s)} \pi(s) + \bar{w}'(s)F(s|s) + \int_s^1 B(t)A(s, t) f(t|s) dt.$$

Noting that

$$\bar{w}(s)F(s|s) = \int_0^s w(s, t) f(t|s) dt,$$

it follows that

$$\bar{w}'(s)F(s|s) = (w(s, s) - \bar{w}(s))f(s|s) + \int_0^s w_1(s, t) f(t|s) dt + \int_0^s w(s, t) A(s, t) f(t|s) dt.$$

Thus, the difference in slopes of the two curves at any $s > a$ is

$$\frac{d\pi(s)}{ds} - \frac{d\pi^{NC}(s)}{ds} = \frac{F_2(s|s)}{F(s|s)} [\pi(s) - \pi^{NC}(s)] + (w(s, s) - \bar{w}(s))f(s|s) + \int_s^1 B(t)A(s, t) f(t|s) dt,$$

where the second term on the right-hand side is positive because $w(s, t)$ is increasing in t and, due to Lemma 2, the third term is positive. It then follows that

$$\frac{d\pi(s)}{ds} - \frac{d\pi^{NC}(s)}{ds} > 0$$

whenever $\pi(s) = \pi^{NC}(s)$.

Q.E.D.

Proof of Lemma 7. Efficiency implies that $Q_1(s_1, s_2) = 1$ if $s_1 > b$ and $s_1 > s_2 > \theta(s_1)$ and equal to 0 otherwise. It then follows from (5) that

$$\frac{d\pi_1(s)}{ds} = \int_{\theta(s)}^s w_1(s, t) dF(t)$$

for $s \geq b$ and 0 otherwise. Integrating the above equation yields

$$\pi_1(s) = \pi_{10} + \int_b^s \int_{\theta(y)}^y w_1(y, t) dF(t) dy$$

for $s \geq b$, where π_{10} is a constant. Changing integration order in the above expression yields

$$\pi_1(s) = \pi_{10} + K(s) - \int_b^s [w(t, t) - r]dF(t),$$

where

$$K(s) = \int_{\theta(s)}^s [w(s, t) - r]dF(t).$$

The profit expression for buyer 2 can be derived symmetrically.

From (4),

$$\pi_1(s) = E_t[Q_1(s, t) \max\{w(s, t) - r, 0\}] + E_t P_1(s, t).$$

It follows that

$$E_t P_1(s, t) = \pi_1(s) - K(s)$$

for $s \geq b$ and is equal to $\pi_1(s)$ if $s \leq b$. Thus,

$$\begin{aligned} E_{(s,t)} P_1(s, t) &= E_s \pi_1(s) - \int_b^1 K(s) dF(s) \\ &= \pi_{10} - \int_b^1 \int_b^s [w(t, t) - r]dF(t) dF(s). \end{aligned}$$

Using integration by parts, we obtain

$$E_{(s,t)} P_1(s, t) = \pi_{10} - \int_b^1 [w(t, t) - r][1 - F(t)]dF(t).$$

Ex ante budget balance implies $E_{(s,t)} \{P_1(s, t) + P_2(t, s)\} = 0$. It follows that

$$\pi_{10} + \pi_{20} = 2 \int_b^1 [w(t, t) - r][1 - F(t)]dF(t).$$

Proof of Corollary 9. First notice that the profit for a buyer with signal s from the seller's auction, $\pi^{NC}(s)$, is equal to zero for $s \leq a$, and strictly increasing in s for $s \geq a$. Moreover, for $s > a$,

$$\frac{d\pi^{NC}(s)}{ds} = \int_0^s w_1(s, x) dF(x).$$

On the other hand, by Lemma 7, the profit for a bidder with signal s from an efficient, incentive ring mechanism, $\pi^C(s)$, is a positive constant when $s \leq b$, and strictly increasing in s for $s \geq b$. Furthermore, for $s > b$,

$$\frac{d\pi^C(s)}{ds} = \int_{\theta(s)}^s w_1(s, x) dF(x).$$

Because $a > b$, it follows that, for any $s > b$,

$$\frac{d\pi^{NC}(s)}{ds} \geq \frac{d\pi^C(s)}{ds}.$$

Therefore, $\pi^C(s) \geq \pi^{NC}(s), \forall s \in [0, 1]$ if and only if $\pi^C(1) \geq \pi^{NC}(1)$. The claim follows. Q.E.D.

Proof of Proposition 10. Let

$$u_1(s, \tilde{s}) = E_t[\max\{w(s, t) - r, 0\}Q_1(\tilde{s}, t) - P_1(\tilde{s}, t)]$$

and

$$u_1(s) = u_1(s, s).$$

$u_2(t)$ and $u_2(t, \tilde{t})$ are similarly defined. An allocation mechanism $[Q_i, P_i]$ is incentive compatible if and only if

$$u_1(s, \tilde{s}) \leq u_1(s)$$

and

$$u_2(t, \tilde{t}) \leq u_2(t)$$

for all $s, t, \tilde{s}, \tilde{t} \in [0, 1]$. Let M be the set of incentive compatible and *ex ante* budget balance allocation mechanisms. We first show that if

$$w_1(s, t) \frac{F(s)}{f(s)} \geq w_1(t, s) \frac{F(t)}{f(t)}$$

if and only if $s \geq t$, then $u_1(1) + u_2(1)$ is maximized in M when

$$Q_1(s, t) = \begin{cases} 1, & \text{if } s > t \\ 1/2, & \text{if } s = t \\ 0, & \text{otherwise} \end{cases}$$

$$Q_2(t, s) = 1 - Q_1(s, t).$$

Let $\Omega(s) = \{t \mid w(s, t) \geq r\}$ and $\tilde{\Omega} = \{(s, t) \mid w(s, t) \geq r\}$. Then necessary and sufficient conditions for IC are

$$(i) u'_1(s) = \int_{\Omega(s)} w_1(s, t) Q_1(s, t) dF(t)$$

and

$$(ii) \int_{\Omega(s)} w_1(s, t) Q_1(\tilde{s}, t) dF(t) \text{ weakly increases with } \tilde{s}$$

for buyer 1 and similar conditions for buyer 2.

Because

$$Q_1(s, t) + Q_2(t, s) = 1,$$

we let $Q(s, t) = Q_1(s, t)$ and $Q_2(t, s) = 1 - Q(s, t)$. Let $u_1(0) = u_{01}$. It follows that

$$\begin{aligned} E_s u_1(s) &= \int_0^1 u_1(s) dF \\ &= u_{01} + \int_0^1 u'_1 [1 - F] ds \\ &= u_{01} + \int_0^1 \int_{\Omega(s)} \left(w_1(s, t) \frac{1 - F(s)}{f(s)} Q(s, t) \right) dF dF. \end{aligned}$$

Because, by definition,

$$u_1(s) = E_t [\max[w(s, t) - r, 0] Q(s, t) - P_1(s, t)],$$

it follows that

$$\begin{aligned} E_{s,t} P_1(s, t) &= E_{s,t} [\max[w(s, t) - r, 0] Q(s, t)] - E_s u(s) \\ &= -u_{01} + \int_0^1 \int_{\Omega(s)} I(s, t) Q(s, t) dF dF \\ &= -u_{01} + \int_{\tilde{\Omega}} I(s, t) Q(s, t) dF dF, \end{aligned}$$

where

$$I(s, t) = w(s, t) - r - w_1(s, t) \frac{1 - F(s)}{f(s)}.$$

Symmetrically, we have

$$E_{t,s} P_2(t, s) = -u_{02} + \int_{\tilde{\Omega}} I(t, s) [1 - Q(s, t)] dF dF.$$

Ex ante budget balance requires that

$$\begin{aligned} 0 &= E_{s,t} P_1(s, t) + E_{s,t} P_2(s, t) \\ &= -u_{01} - u_{02} + \int_{\tilde{\Omega}} (I(t, s) + [I(s, t) - I(t, s)] Q(s, t)) dF dF. \end{aligned}$$

It follows that

$$u_{01} + u_{02} = \int_{\tilde{\Omega}} (I(t, s) + [I(s, t) - I(t, s)] Q(s, t)) dF dF.$$

Thus,

$$\begin{aligned} u_1(1) + u_2(1) &= \int_{\tilde{\Omega}} (I(t, s) + [I(s, t) - I(t, s)] Q(s, t)) dF dF \\ &\quad + \int_{\tilde{\Omega}} \frac{w_1(s, t)}{f(s)} Q(s, t) dF dF + \int_{\tilde{\Omega}} \frac{w_1(t, s)}{f(t)} [1 - Q(s, t)] dF dF \\ &= \int_{\tilde{\Omega}} I(t, s) dF dF + \int_{\tilde{\Omega}} [J(s, t) - J(t, s)] Q(s, t) dF dF, \end{aligned}$$

where

$$J(s, t) = w(s, t) - r + w_1(s, t) \frac{F(s)}{f(s)}.$$

Because $w(s, t) - r = w(t, s) - r$, it follows that

$$J(s, t) - J(t, s) = w_1(s, t) \frac{F(s)}{f(s)} - w_1(t, s) \frac{F(t)}{f(t)}.$$

It follows from the assumption that $J(s, t) - J(t, s) \geq 0$ if and only if $s \geq t$. Therefore, $u_1(1) + u_2(1)$ is maximized in M when the efficient allocation rule is used.

Because the first-price knockout auction implements the efficient allocation rule, to check whether the interim participation constraints are violated it is without loss of generality to compare the payoff from the knockout auction and that from the competitive bidding at the highest signal. The claim follows. *Q.E.D.*

Proof of Proposition 11. The proof is by contradiction. Recall that all types make strictly positive profits in the ring mechanism. If it is not ratifiable, then there exists a buyer i , a veto set V_i , and an equilibrium in the continuation game such that all types in V_i earn strictly positive profits. Let G_i denote the equilibrium bid distribution in the continuation game and let \underline{b}_i denote the lower bound of this distribution. Let R_j denote the set of bidder j types that bid \underline{b}_i or less with positive probability. Let K_i denote the set of veto types that bid \underline{b}_i . Because all types earn positive profits in the collusive mechanism,

$$\pi_i(s', \underline{b}_i) = E[V - \underline{b}_i | S_i = s', s_j \in R_j] \Pr\{s_j \in R_j | S_i = s'\} > 0,$$

for all $s' \in K_i$. The strict inequality implies (i) $\underline{b}_i \geq r$ and (ii) the existence of a nondegenerate set of bidder j types who bid \underline{b}_i or less. If $G_i(\underline{b}_i) = 0$, then R_j consists of bidder j types that are certain to lose in the *status quo* mechanism. It then follows from pure common values that there exists types $t_j \in R_j$ such that, for a sufficiently small $\epsilon > 0$,

$$E[V - \underline{b}_i - \epsilon | S_j = t_j, s_i \in K_i] > 0.$$

That is, the expected payoff to t_j from bidding slightly more than \underline{b}_i is positive. This profitable deviation contradicts the claim that veto types bidding \underline{b}_i make positive profits.

If $G_i(\underline{b}_i) > 0$, then R_j consists of bidder j types who bid strictly less than \underline{b}_i because otherwise the veto types bidding \underline{b}_i could increase their profits by slightly more. Thus, R_j consists of bidder j types who are certain to lose and the argument given above applies. *Q.E.D.*

Appendix B

■ In this appendix, we present the expressions for payoffs and cutoff points used in Section 3.

□ **B1. Status Quo Mechanism.** The payoff for a buyer of type s is given by

$$\pi^{NC}(s) = \frac{1}{1+q} (s^{1+q} - a^{1+q})$$

for $s \geq a$ and zero otherwise, where

$$a = \frac{1+q}{1+2q} r.$$

□ **B2. Knockout Auction.** A first-price knockout auction generates the following profits to a buyer with signal s :

$$\pi^K(s) = \pi_0 + \int_{\theta(s)}^s (s+x-r) dx^q - \int_b^s (2x-r) dx^q$$

for $s > b = r/2$ and is equal to π_0 otherwise, where

$$\pi_0 = \int_b^1 (2x-r)(1-x^q) dx^q$$

and $\theta(s) = \max\{r-s, 0\}$. Note that

$$\pi^K(1) = \pi_0 + \int_{\theta(1)}^1 (1+x-r) dx^q - \int_b^1 (2x-r) dx^q,$$

where

$$\begin{aligned}\pi_0 &= \int_b^1 (2x - r)(1 - x^q) dx^q \\ &= \frac{2q^2}{(1+q)(1+2q)} - b + \frac{2}{1+q}b^{1+q} - \frac{1}{1+2q}b^{1+2q}, \\ \int_{\theta(1)}^1 (1+x-r) dx^q &= 2-r - \frac{1}{1+q} + \frac{1}{1+q}\theta(1)^{1+q} \\ \int_b^1 (2x-r) dx^q &= 2-r - 2 \int_b^1 x^q dx \\ &= 2-r - \frac{2}{1+q} + \frac{2}{1+q}b^{1+q}.\end{aligned}$$

Thus,

$$\pi^K(1) = \frac{2q^2 + 2q + 1}{(1+q)(1+2q)} - \frac{r}{2} - \frac{1}{1+2q} \left(\frac{r}{2}\right)^{1+2q} + \frac{1}{1+q} \max\{r-1, 0\}^{1+q}.$$

□ **B3. Equal Sharing Mechanism.** The payoff for a buyer with signal s is given by

$$\pi^E(s) = \frac{1}{2} \int_{\theta(s)}^1 (s+x-r) dx^q,$$

which can be simplified as follows:

$$\pi^E(s) = \frac{q}{2(1+q)} - \frac{r}{2} + \frac{s}{2} + \frac{\theta(s)^{1+q}}{2(1+q)}$$

for all $s \in [\theta(1), 1]$ and 0 otherwise.

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